IX. Electroweak unification

The problem of divergence

- A theory of weak interactions only by means of $W^{\pm}$ bosons leads to infinities

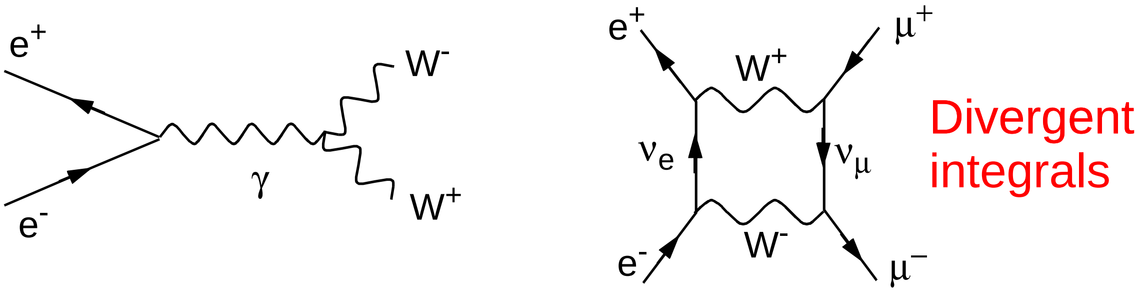


Figure 108: Examples of divergent processes.

→ Introduction of the Z^0 boson fixes the problem because the addition of new diagrams cancel out the divergencies:

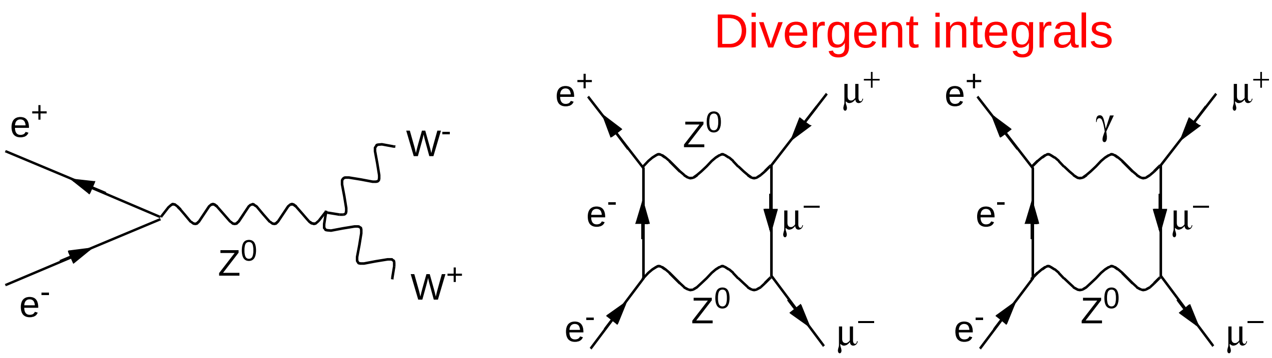


Figure 109: Additional processes which cancel the divergence.

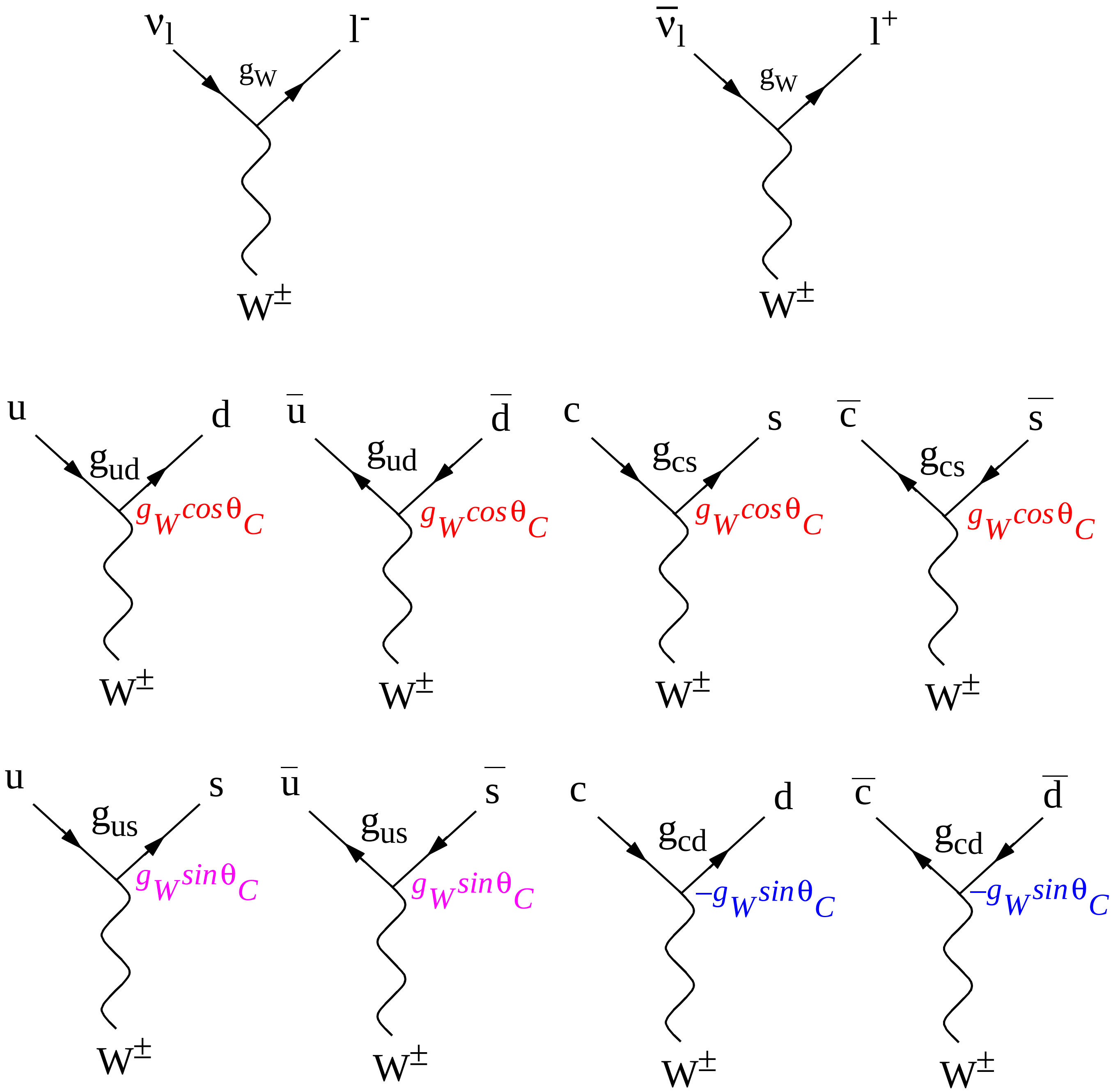


Figure 110: REMINDER: The basic W lepton and quark vertices (if the third generation is not taken into account).

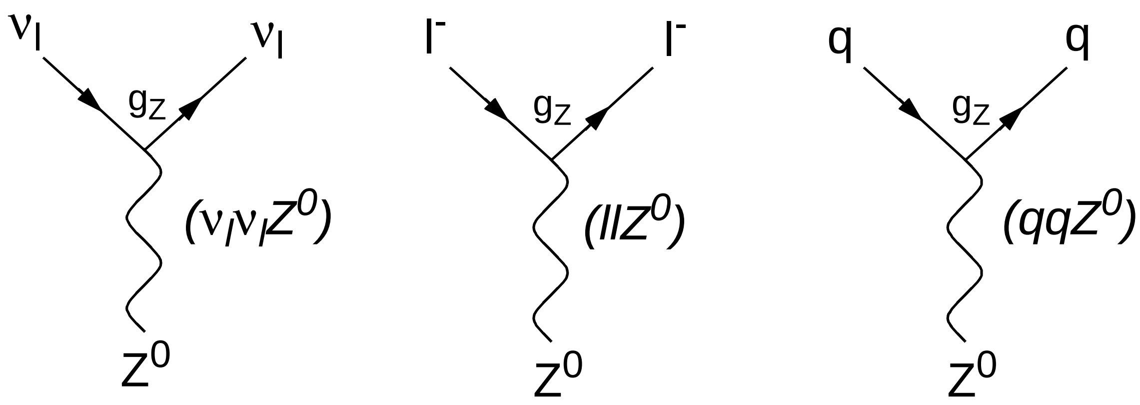


Figure 111: The basic Z^0 lepton and quark vertices.

- Basic vertices with W bosons have:

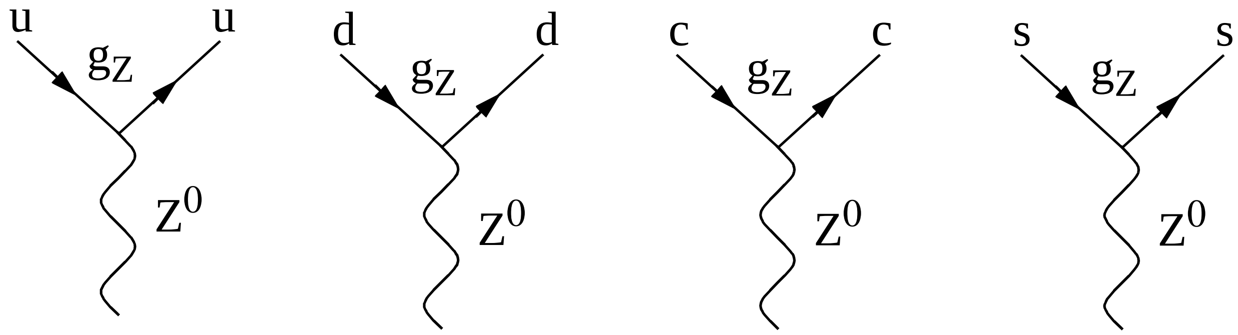
– Conserved lepton numbers

– Not conserved quark flavour (quark mixing)

- Basic vertices with Z^0 bosons have:

– Conserved lepton numbers

– Conserved flavour (no quark mixing)



Test of flavour conservation

Flavour is conserved at a Z^0 vertex (in contrast to a W vertex).This can be verified by experiments.

Consider the following two possible processes that change strangeness:

$$K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu \qquad \text{(a)}$$

and

$$K^+ \rightarrow \pi^+ + \nu_l + \bar{\nu}_l \qquad \text{(b)}$$

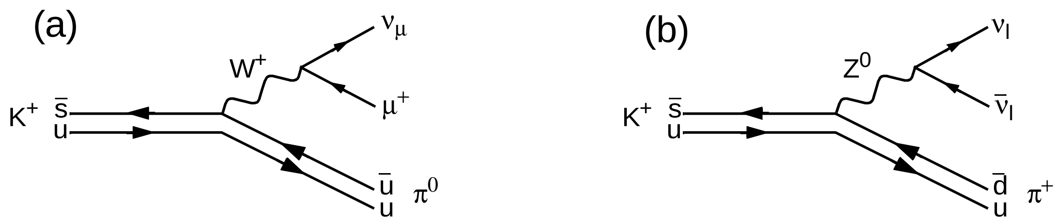


Figure 112: Decay (a) is allowed; decay (b) – forbidden

The measured upper limit on the ratio of the decay rates (b) to (a) is:

$$\frac{\sum_l \Gamma(K^+ \rightarrow \pi^+ + \nu_l + \nu_l)}{\Gamma(K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu)} < 10^{-7}$$

The unification condition and masses

The coupling constants at γ–, $W^{\pm}$– and Z^0-vertices are not independent from each other. In order for all infinities to cancel in electroweak theory, the unification relation and the anomaly condition have to be fulfilled.

→ The *unification condition* establishes a relation between the coupling constants ($\alpha_{em}=e^2/4\pi\varepsilon_0$):

$$\sqrt{\frac{\pi \cdot \alpha}{2}} = g_W \sin\theta_W = g_Z \cos\theta_W \qquad (114)$$

θ_W is the *weak mixing angle*, or *Weinberg angle*:

$$\cos\theta_W = \frac{M_W}{M_Z} \qquad (115)$$

→ The *anomaly condition* relates electric charges: $\sum_l Q_l + 3\sum_q Q_q = 0$

where the factor 3 comes from the number of colors.

Historically, the W and Z masses were predicted from low energy interactions.

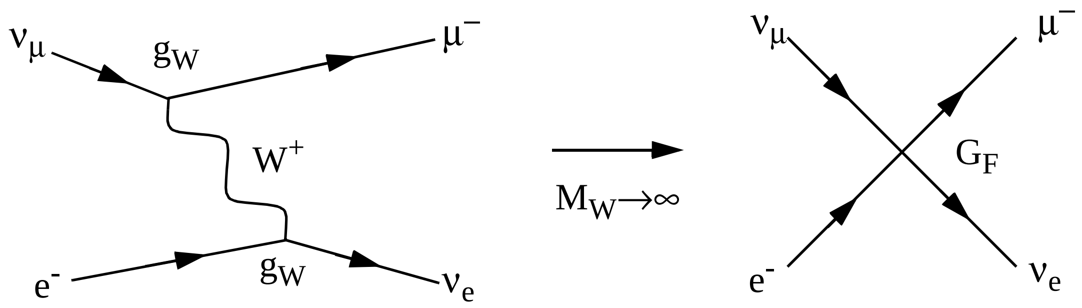


Figure 113: The low energy zero range approximation.

In the zero-range approximation i.e. in the low-energy limit, the charged current reactions are characterized by the Fermi constant (G_F):

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2}$$

From this expression, the unification condition and the definition of θ_W one then obtains:

$$M_W^2 = \frac{g_W^2\sqrt{2}}{G_F} = \frac{\pi\alpha}{\sqrt{2}G_F sin^2\theta_W}$$

$$M_Z^2 = \frac{\pi\alpha}{\sqrt{2}G_F cos^2\theta_W sin^2\theta_W}$$

Introducing the neutral current coupling constant (G_Z) (also in the low energy zero-range approximation) one gets

$$\frac{G_Z}{\sqrt{2}} = \frac{g_Z^2}{M_Z^2}$$

and the weak mixing angle can be expressed as

$$\frac{G_Z}{G_F} = \frac{g_Z^2 M_W^2}{g_W^2 M_Z^2} = \sin^2\theta_W$$

From the measurements at low energy of rates of charged and neutral currents reactions it is therefore possible to determine that:

$$\sin^2\theta_W = 0,277 \pm 0,014$$

from this measurement at low energies (below the W and Z masses) it was possible to predict the masses of W and Z:

$$M_W = 78,3 \pm 2,4\ GeV/c^2;\ M_Z = 89,0 \pm 2,0\ GeV/c^2$$

When the W and Z boson were discovered at CERN with the masses predicted from low energy experiments it was a strong confirmation that the electroweak theory was correct.

The search for the Higgs boson

- The mass of the Higgs itself is not predicted by the theory, only the couplings to other particles.
- The existence of the Higgs boson has not been confirmed by experiments.

Searches for the Higgs at LEP.

a) If the H^0 was lighter than the Z^0 ($M_H \leq 60$ GeV), then the Z^0 could decay by

$$Z^0 \to H^0 + l^+ + l^- \qquad (121)$$

$$Z^0 \to H^0 + \nu_l + \bar{\nu}_l \qquad (122)$$

But the branching ratio is very low:

$$3\times 10^{-6} \leq \frac{\Gamma(Z^0 \to H^0 l^+ l^-)}{\Gamma_{tot}} \leq 10^{-4}$$

The measurements at LEP 1 has set a *lower limit* on the Higgs mass which is $M_H > 58$ GeV/c^2

b) If the H^0 is heavier than 60 GeV/c^2, it could have been produced in e$^+$e$^-$ annihilations at LEP 2. The most important process is:

$$e^+ + e^- \rightarrow H^0 + Z^0 \quad (123)$$

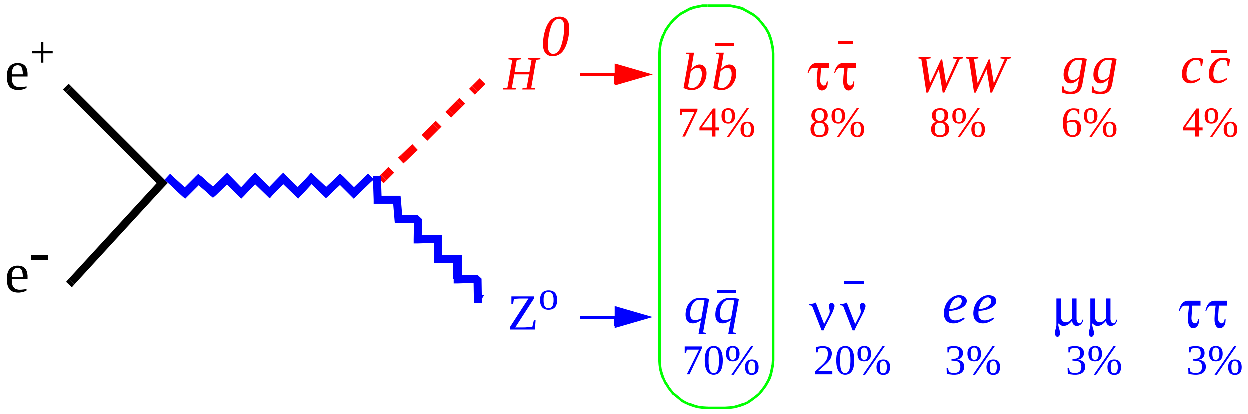


Figure 122: "Higgsstrahlung" in e$^+$e$^-$ annihilation

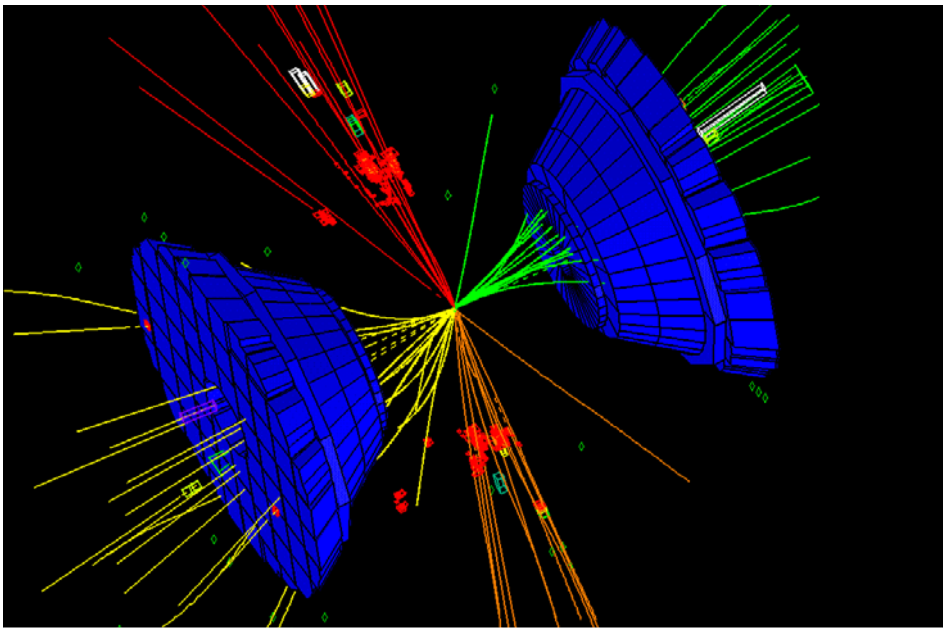


Figure 123: Example of a Higgs candidate event (Delphi).

❖ During the last year of operation of LEP 2, the ALEPH experiment recorded a couple of events which could be due to the decays of a Higgs with a mass of about 115 GeV/c^2. The other LEP experiments could not confirm the ALEPH results and the DELPHI experiment set a limit of:

$$M_H > 114\ GeV/c^2$$

The measurement of many electroweak parameters at LEP (and other places) makes it possible to make a global fit with the Higgs mass as a free parameter

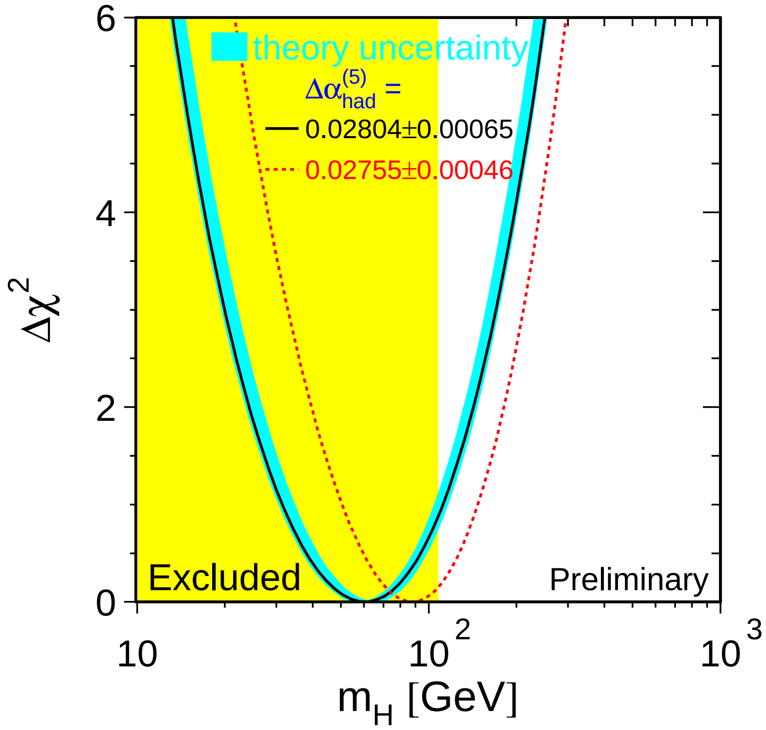


Figure 124: A prediction of the Higgs mass from a global fit to electroweak measurements.

❖ The result of the fit is a prediction of a low mass for the Higgs boson < 165 GeV.

Searches for the Higgs at LHC.

c) Higgs with masses up to 1 TeV can be observed at the future proton-proton collider LHC at CERN:

$$p + p \rightarrow H^0 + X \qquad (124)$$

where H^0 is produced in electroweak interaction between the quarks

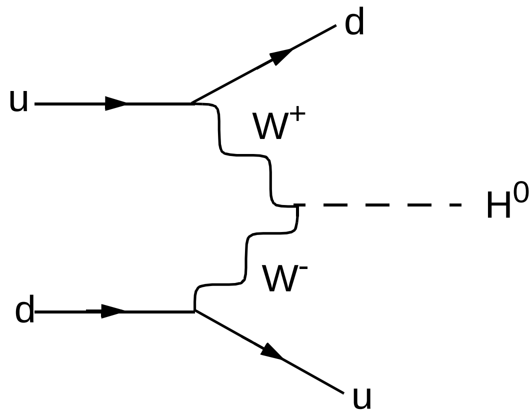


Figure 125: An example of Higgs production process at LHC

At the LHC the background is huge and a good signature have to be found.

– If $M_H < 2M_W$, (160 GeV/c[2]) the dominant decay mode is

$$H^0 \rightarrow b + \bar{b} \qquad (125)$$

but these events will be swamped by background. A more promising decay mode is

$$H^0 \rightarrow \gamma + \gamma \qquad (126)$$

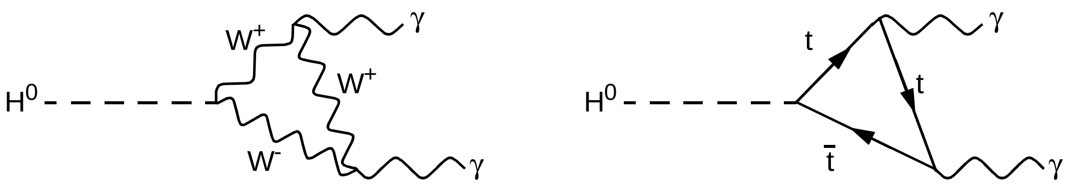


Figure 126: The dominant mechanisms for the decay to photons

The branching ratio of this kind of processes is, however, only 10^{-3}

– If $M_H > 2M_Z$, the dominant decay modes are:

$$H^0 \rightarrow Z^0 + Z^0 \quad (127)$$

$$H^0 \rightarrow W^- + W^+ \quad (128)$$

The most clear signal is when both Z^0s decay into electron or muon pairs:

$$H^0 \rightarrow l^+ + l^- + l^+ + l^- \quad (129)$$

These decays can be found if $200 \leq M_H \leq 600$ GeV, but only 4% of all Higgs particles decay to four electrons or muons.

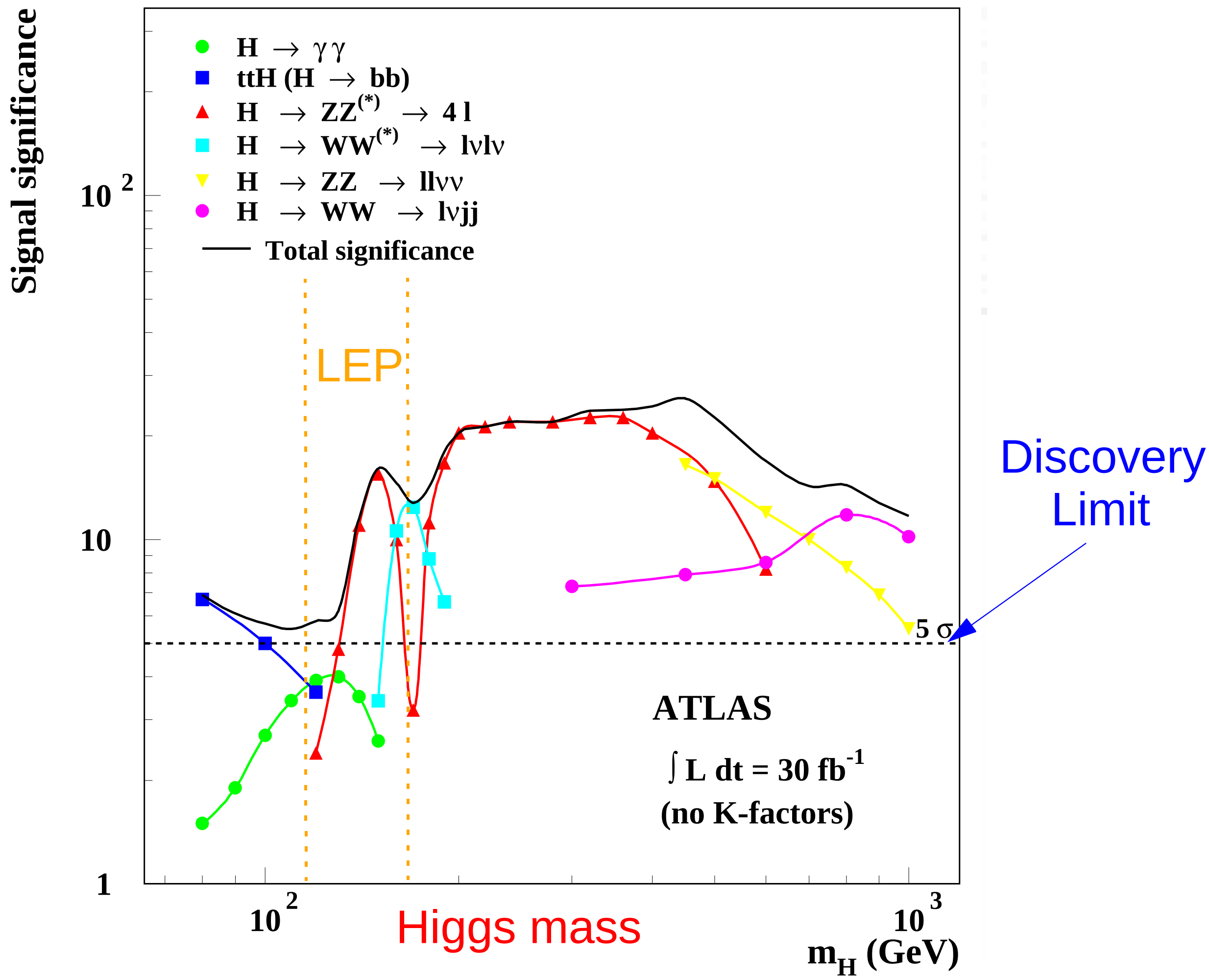


Figure 127: Higgs discovery potenial at the LHC.

Summary

- **The problem of divergence**

  a) By introducing the Z-bosons one can cancel out divergent diagrams from the W-bosons.

  b) There is no quark mixing in Z-vertices.

- **Test of flavour conservation.**

  c) Kaon decay show that flavour is conserved at a Z-vertex (but not a W-vertex).

- **The unification condition and masses.**

  d) The unification condition establishes a relation between the electromagnetic coupling constants.

  e) The ratio of the W- and Z-masses is given by the weak mixing angle (the Weinberg angle).

- **Electroweak reactions**

  f) Fitting the Z-peak gives the mass and width of the Z-boson. From this, it can be determined that the number of light neutrino families is 3.

• Gauge invariance.

g) A gauge transformation is a symmetry transformation.

h) Field theories which do not change under gauge transformation are gauge invariant.

i) Imposing gauge invariance on the weak interaction theory leads to the prediction of three massless W-bosons.

j) The unification of electromagnetism with weak interactions leads to the introduction of the B^0-boson which is connected to the electromagnetic field.

k) The neutral gauge bosons that are observed in experiments (γ and Z^0) are mixtures of the B^0 and W^0 states.

• The Higgs boson.

l) The Higgs field and its gauge boson are introduced to explain the large masses of the W- and Z-bosons.

m) The Higgs field has the unusual feature of having a non-zero expectation value in vacuum.

- **The search for the Higgs boson**

n) The LEP experiments have been the main place for the search for a Higgs up to now.

o) In the future the search will take place at the Tevatron followed by the LHC.